

## **Discussion Comments on “Momentum and Autocorrelation in Stock Returns”**

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Jegadeesh and Titman (1993) document individual stock momentum: strategies that buy stocks that have performed relatively well in the past and sell stocks that have performed relatively poorly in the past generate significant positive returns over the 3- to 12-month horizon. This finding, obtained using data from the US market, also holds for a number of international markets (e.g., Haugen and Baker (1996), Rouwenhorst (1998)). What are the economic mechanisms behind individual stock momentum?

One approach to answering this question is to relate momentum to other factors driving the cross-section of expected stock returns. A number of findings have emerged. Adjustments for factors such as the Fama-French three-factor model tend to strengthen, rather than explain, momentum (e.g., Jegadeesh and Titman (1993), Fama-French (1996), Grundy and Martin (2001)). Contrary to the finding of Conrad and Kaul (1998), cross-sectional differences in expected returns is not an important cause of momentum (e.g., Grundy and Martin (2001), Jegadeesh and Titman (2001)). And contrary to the finding of Moskowitz and Grinblatt (1999), a number of subsequent papers find that industries (or industry effects) do not explain momentum (e.g., Grundy and Martin (2001), Asness, Porter and Stevens (2000), Lee and Swaminathan (2000)). Stock price momentum is partially related to earnings momentum, but both past returns and public earnings surprises (in a multiple regression) help to predict subsequent returns at horizons of six months to a year (e.g., Chan, Jegadeesh and Lakonishok (1996)).

### **I. The Paper’s Main Findings and Conclusions**

Following this approach, this provocative paper has two sets of findings. The first set is similar to the finding of industry momentum (Moskowitz and Grinblatt (1999))---for a set of industry portfolios, buying past winning portfolios and selling past losing portfolios generate positive returns for horizons out to about one year. Lewellen finds that size and book-to-market portfolios also exhibit momentum, similar in magnitude but distinct from momentum in individual stocks and industry. These new findings add to the growing list of stylized facts

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regarding stock price momentum. Like industry momentum, size and book-to-market momentum can conceivably be consistent with any number of explanations.<sup>1</sup>

The second set of findings is based on Lo and MacKinlay (1990), who observe that the expected profit of a momentum strategy might come from three different sources. A stock that performed well relative to other stocks might continue to do so because: (1) the stock simply has a high unconditional mean relative to other stocks (this source has already been rejected in the literature), (2) the stock return is positively correlated, so its own past return predicts high future returns, and (3) the stock return is negative correlated with lagged returns on other stocks (negative cross-serial covariance), so their poor performance predict high future returns.

Using raw returns, Lewellen calculates (separately) for industry, size and book-to-market portfolios the auto- and cross-serial covariances among the portfolios. For each of these three sets of portfolios, the average of the auto-covariances is slightly negative but is not statistically significant. The corresponding average of the cross-serial covariances tend to be more negative but are also not generally statistically significant.

He draws two conclusions from these findings. First, the negative average of the auto-covariances is evidence that momentum in industry, size and book-market portfolios are not due to past winners (losers) continuing to be winners (losers). So, this is inconsistent with the under-reaction based, behavioral models of momentum such as Barberis, Shleifer and Vishny (1998) and Hong and Stein (1999), which imply positive auto-covariances in stock returns. Second, the momentum in these portfolios is due to future stock return being negatively correlated with the lagged return of other stocks (negative cross-serial covariance), consistent with an over-reaction hypothesis in which certain stocks overreact to a common factor and others do not.

## **II. Our Main Points**

Beyond the obvious issues with the statistical significance of the auto- and cross-serial covariance estimates, we argue that the two conclusions of the paper are unwarranted. First, we show with a simple example that his empirical findings can in general be consistent with momentum being due to the under-reaction to shocks. In other words, it does not in general follow that just because raw auto- and cross-serial covariances are on average negative that under-reaction is not the source of momentum. Second, using an insight from this example, we

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<sup>1</sup> In a world with individual stock price momentum, one would expect that even portfolios created randomly might exhibit some momentum. Grundy and Martin (2001) find that this is indeed the case. And depending on how the industry momentum strategy is constructed, they find that the difference between industry momentum and momentum in random portfolios is not statistically significant.

show that the industry, size and book-to-market momentum are not likely to be due to the over-reaction hypothesis advocated by the paper. Instead, these findings are more consistent with under-reaction based explanations.

### III. A Simple Example

Consider a one-factor world in which the only source of momentum is under-reaction to shocks. Assume there are  $N$  stock returns (think of them as  $N$  industry portfolios) with the following one-factor structure:

$$\begin{aligned} f_t &= \rho f_{t-1} + e_t, \\ r_{i,t} &= \mu_i + \beta_i f_t + \varepsilon_{i,t}, \end{aligned} \tag{1}$$

where  $e_t$  is a mean-zero serially uncorrelated shock to the factor  $f_t$  and  $\varepsilon_{i,t}$  is a mean-zero, positively serially correlated idiosyncratic shock with variance and  $E[\varepsilon_{i,t}\varepsilon_{i,t-1}] = \kappa\sigma_\varepsilon^2 > 0$ . The unconditional variance of the factor  $f_t$  is  $\sigma_f^2$ . (Think of  $f_t$  as the demeaned market factor.) The parameter  $\rho$  is the serial correlation of the factor. All other correlations, serial correlations and cross-serial correlations are assumed to be zero. And assume that every asset has the same mean and beta:  $\mu_i = \mu$  and  $\beta_i = \beta = 1$ . By construction, the only source of momentum profit in this setting is the positive serial correlation of the idiosyncratic (e.g. industry) shock, which one can think of as due to some under-reaction mechanism along the lines of recent behavioral models.

Lets verify this by calculating the expected momentum profit. Recall that a momentum strategy buys stocks based on their returns in period  $t-1$  and holds the stocks in period  $t$ . With this strategy, the portfolio weight assigned to stock  $i$  at time  $t$  is

$$w_{i,t} = \frac{1}{N}(r_{i,t-1} - r_{t-1}), \tag{2}$$

where  $r_t = \frac{1}{N} \sum_{i=1}^N r_{i,t}$  is the equally weighted index return and  $\bar{\mu}$  the expected return of the index. By construction, the total investment at any given time is zero. However, the dollar investments in the long and short sides of the portfolio vary over time depending on the return realizations at time  $t-1$ . The time  $t$  profit of this momentum strategy is

$$\pi_t = \frac{1}{N} \sum_{i=1}^N w_{i,t} r_{i,t}. \tag{3}$$

Given the one-factor structure in (1), the momentum strategy in (2) is given by weights in each asset of

$$w_{i,t} = \frac{N-1}{N^2} \varepsilon_{i,t-1} - \frac{1}{N^2} \sum_{j \neq i}^N \varepsilon_{j,t-1}. \quad (4)$$

Therefore, the momentum profit in (3) is simply

$$\pi_t = \sum_{i=1}^N \left[ \frac{N-1}{N^2} \varepsilon_{i,t-1} - \frac{1}{N^2} \sum_{j \neq i}^N \varepsilon_{j,t-1} \right] r_{i,t}. \quad (5)$$

It is easy to show that the expected momentum profit is

$$E[\pi_t] = \frac{N-1}{N} \kappa \sigma_\varepsilon^2. \quad (6)$$

By construction, it only depends on the positive auto-covariance of the idiosyncratic shocks since the unconditional means and stock betas are assumed to be identical.

Now, let's calculate the Lo-MacKinlay decomposition for our one-factor example. Recall from Lo and MacKinlay (1990) that expected momentum profit is given by the following mathematical identity:

$$E[\pi_t] \equiv \sigma_\mu^2 + O - C, \quad (7)$$

where the first term  $\sigma_\mu^2$  is the cross-sectional variance of expected returns given by

$$\sigma_\mu^2 = \frac{1}{N} \sum_{i=1}^N (\mu_i - \bar{\mu})^2, \quad (8)$$

the second term  $O$  is the average auto-covariance of raw returns given by

$$O = \frac{N-1}{N^2} \sum_{i=1}^N E(r_{i,t} r_{i,t-1} - \mu_i^2), \quad (9)$$

and the third term  $C$  is the average cross-serial covariance given by

$$C = E(r_t r_{t-1}) - \bar{\mu}^2 - \frac{1}{N^2} \sum_{i=1}^N E(r_{i,t} r_{i,t-1} - \mu_i^2). \quad (10)$$

Now, applying the formulas in (8)-(10) to our one-factor example, it is easy to show that  $\sigma_\mu^2 = 0$  (as expected), that

$$O = \frac{N-1}{N} \rho \sigma_f^2 + \frac{N-1}{N} \kappa \sigma_\varepsilon^2, \quad (11)$$

and that

$$C = \frac{N-1}{N} \rho \sigma_f^2. \quad (12)$$

Notice that  $\sigma_\mu^2 + O - C = \frac{N-1}{N} \kappa \sigma_\varepsilon^2$ , which is simply the expected momentum profit calculated above in (6).

When the common factor is serially correlated ( $\rho \neq 0$ ), the average auto-covariance  $O$  naturally depends on the auto-covariances of the idiosyncratic shocks and the common factor. The expected momentum profit  $E[\pi]$  depends *only* on the auto-covariance of the idiosyncratic shocks (and not of the common factor) by construction. Hence, the cross-serial covariance term  $C$  has to be such that the identity given in (7), the Lo-MacKinlay decomposition, holds.

So even though momentum profits do not depend on  $\rho$ , the Lo-MacKinlay decomposition does. It yields negative auto- and cross-serial covariances or positive ones depending on the serial correlation of the factor during the sample period over which the decomposition is done. From (11) and (12), if the auto-correlation of the common factor is positive, then  $O$  and  $C$  are positive. If it is sufficiently negative, then both  $O$  and  $C$  are negative.

We draw two conclusions from this simple example. First, the Lo-MacKinlay decomposition is not informative in general about the underlying economic source of momentum profit. Second, momentum due to under-reaction can be entirely consistent with negative auto- and cross-serial covariances.

In our Table 1, we show that the findings of the paper are likely to be driven by the serial correlation of the market factor in sample. In Panel A, we calculate the momentum profits using a set of 20 industry portfolios based on the strategy given in (2) using past six-month returns. The numbers are 0.072% semi-annual during the 1941-1999 and 0.064% semi-annual during 1928-1999. They are similar to the ones reported in the paper. The momentum profits in the two periods are roughly the same. We also report the serial correlation in the market factor (value-weighted CRSP) using six-month returns. The numbers are  $-0.04$  during the 1941-1999 and  $0.08$  during 1928-1999. In Panel B, we report the corresponding the Lo-MacKinlay decomposition given by (7)-(10). During the 1941-1999, the cross-serial covariance term  $C$  is negative and the auto-covariance term  $O$  is also negative. During the 1928-1999,  $C$  is now positive and  $O$  is positive as well, just as our one-factor example predicts.

In fact, we have replicated some of the findings on cross-serial covariances (measured at different horizons) reported in the paper's Table 6. We find that these magnitudes are similar to the values predicted by equation (12). In other words, it appears that a sizeable fraction of the

paper's findings is due to the in-sample negative serial correlation of the market as opposed to a over-reaction mechanism advocated by the paper.

#### IV. An Alternative Decomposition of Expected Momentum Profit

As the simple example clearly shows, the Lo-MacKinlay decomposition is not in general informative about the economic sources of momentum, resulting in potentially misleading inferences. How then do we go about testing an over-reaction based explanation of momentum? Jegadeesh and Titman (1995) provides an answer to this question. Suppose that stocks follow the following factor structure:

$$r_{i,t} = \mu_i + b_{0,i} f_t + b_{1,i} f_{t-1} + \varepsilon_{i,t}, \quad (13)$$

where  $f_t$  is the unexpected factor realization,  $\varepsilon_{i,t}$  is the asset specific component of return at time  $t$  and  $(b_{0,i}, b_{1,i})$  are the time-invariant sensitivities of stock  $i$  to the contemporaneous and lagged realizations of the factor.

This model is similar to conventional multifactor models except that it allows for the lagged factor sensitivities to be different from zero. To see how this specification will generate momentum due to over-reaction, imagine that there are only two industries A and B, with say  $(b_{0,A} = 1.3, b_{1,A} = 0)$  and  $(b_{0,B} = 0.9, b_{1,B} = -0.1)$ . In this case, a naïve strategy of buying the winning industry and selling the losing industry is profitable because of cross-serial covariances due to the over-reaction of industry B. Suppose that there is a positive factor realization, so A is the winning industry. Then buying industry A and selling industry B is profitable since industry B over-reacts to the positive factor shock (and has negative future returns on average). Suppose that there is a negative factor shock, so A is the losing industry. Then the strategy calls for selling industry A and buying B. This is profitable because B overreacts to the negative factor shock (and has positive future returns on average).

The expected momentum profits under (13) is shown by Jegadeesh and Titman (1995) to be given by

$$E[\pi_t] = \sigma_\mu^2 + \Omega + \delta \sigma_f^2, \quad (14)$$

where the first term  $\sigma_\mu^2$  is the cross-sectional variance of expected returns given in (8), the second term  $\Omega$  is the average idiosyncratic auto-covariance given by

$$\Omega = \frac{N-1}{N^2} \sum_{i=1}^N \text{cov}(\varepsilon_{i,t}, \varepsilon_{i,t-1}), \quad (15)$$

and the third term  $\delta$  is the contribution to momentum profits from the differential over-reaction to the common factor given by

$$\delta = \frac{1}{N} \sum_{i=1}^N (b_{0,i} - \bar{b}_0)(b_{1,i} - \bar{b}_1), \quad (16)$$

where  $\bar{b}_{0,i}$  and  $\bar{b}_{1,i}$  are the averages of  $b_{0,i}$ 's and  $b_{1,i}$ 's, respectively. In the context of the two-industry example given above, it is easy to see that  $\delta > 0$ . If over-reaction is a significant contribution for momentum, then we ought to see this latter term contributing to a significant amount to the total momentum profit.

We perform this Jegadeesh and Titman decomposition for the portfolios in our Table 1. For each of the twenty time series of industry portfolios, we estimate equation (14) using the (demeaned) value-weighted market CRSP index as the proxy for the common factor. This estimation yields estimates for  $(\mu_i, b_{0,i}, b_{1,i})$  for each of the twenty time series. With these estimates, we can calculate the residuals  $\varepsilon_{i,t}$  for each of the twenty industry portfolios. This allows us to then calculate the auto-covariance for these time series of residuals and to average these auto-covariances to get an estimate of  $\Omega$ . It is also easy to estimate  $\delta$  and  $\sigma_f^2$ .

Panel C of Table 1 reports the results of this decomposition for 1928-1999 and the 1941-1999 time periods. Notice that in both instances,  $\Omega > 0$  and is responsible for all the momentum profits, whereas the cross-serial covariance term  $\delta\sigma_f^2$  is nearly zero. In other words, the over-reaction model of (14) is not likely to be driving momentum profits, consistent with our earlier finding that the in sample serial correlation of the market likely to be responsible for the observed negative cross-serial covariances. Moreover, Panel C confirms that momentum is due to price continuation, consistent with under-reaction based explanations.

## V. Conclusion

This paper contributes to the literature by documenting the existence of momentum in size and book-to-market portfolios. However, the conclusions that it draws from the Lo-MacKinlay decomposition of expected momentum profit are unwarranted. We show that the paper's findings of negative average auto- and cross-serial covariances among the portfolios can be consistent with under-reaction based explanations. We also show that these findings are driven by the in-sample serial correlation of the market factor and not the over-reaction mechanism advocated by the paper. Our analysis also suggests that doing such decompositions are not likely to further advance our thinking on the economic sources of momentum.

Rather, we ought to develop stock price models that not only fit various stock price predictability patterns such as momentum, but also generate new and testable predictions (see, e.g., Barberis, Shleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998) and Hong and Stein (1999)). While these models are far from being able to simultaneously reconcile all the behaviors of the time series of the market and the cross-section of stock returns, they have yielded new insights into the sources of individual stock price momentum. For instance, Hong, Lim and Stein (2000) find evidence consistent with the gradual-diffusion-of-information hypothesis of Hong and Stein (1999). They find that the profitability of momentum strategies declines sharply with firm size and (controlling for size) is much more profitable for firms with little analyst coverage. Moreover, the effect of coverage also matters more for stocks that are past losers than for past winners. Other empirical papers motivated by these theories include Lee and Swaminathan (2000) who find that momentum is more pronounced in high volume firms than low volume firms and Jegadeesh and Titman (2001) who find momentum profits get reversed at long horizon of 3-5 years. More work along this line needs to be done.

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**Table 1: Momentum Profits and Auto- and Cross-serial Covariances**

Value-weighted industry portfolios are formed using all common U.S. stocks listed on NYSE, AMEX, and NASDAQ. The industry classifications are based on two-digit SIC codes following Moskowitz and Grinblatt (1999) and produce 20 industries: mining, food, apparel, paper, chemical, petroleum, construction, primary metals, fabricated metals, machinery, electrical equipment, transportation equipment, manufacturing, railroads, other transportation, utilities, department stores, retail, financials, and others. Returns are cumulated every 6-months over January through June and over July through December. Momentum strategies are formed every January and July based on prior 6-months returns. The strategies are given in equation (2). Panel A reports the momentum profits expressed as semi-annual returns and the serial correlation estimate of the market factor (value-weighted CRSP),  $\rho$ , calculated using six-month return. Panel B reports the Lo and MacKinlay decomposition of expected momentum profit given in equation (7). Panel C reports the Jegadeesh-Titman decomposition of expected momentum profit given in equation (14). Due to estimation error, this decomposition does not always add up to the momentum profit.

Panel A: Momentum Profits and Serial Correlation in the Market

Sample	Mean Return	SD	t-statistics	$\rho$
1941-1999	0.072%	0.32%	2.44	-0.041
1928-1999	0.064%	0.38%	2.02	0.080

Panel B: Lo-MacKinlay Decomposition

Sample	$\sigma_{\mu}^2$	$O$	$C$	Total
1941-1999	0.004%	-0.065%	-0.133%	0.072%
1928-1999	0.004%	0.209%	0.149%	0.064%

Panel C: Jegadeesh-Titman Decomposition

Sample	$\sigma_{\mu}^2$	$\Omega$	$\delta\sigma_f^2$	Total
1941-1999	0.004%	0.094%	-0.001%	0.090%
1928-1999	0.004%	0.067%	0.015%	0.086%